2011 General HSC solutions

Q23

Income at breakeven point is $20 000.
(Breakeven is where the lines cross ie where cost equals income.)
Income when 500 attend is $50 000.
The difference between these is:
$50 000 - $20 000 = $30 000.

21 (A) First train travels for 2 h at 90 km/h.
Distance travelled is $2 \times 90 = 180$ km.
Second train:
Time taken is from 3:10 pm to 4:30 pm
i.e. 1 h 20 min (or $1\frac{1}{3}$ h).
Distance travelled is 180 km.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{180\text{ km}}{1\frac{1}{3}\text{ h}} = 135\text{ km/h}$$

22 (A) To complete Month 3:

$$P + I - R = \$251\,032.04 - \$1871.94 = \$249\,160.10$$

Month 4: $P = \$249\,160.10$

$$I = Prn = \$249\,160.10 \times \frac{0.0765}{12} \times 1 = \$1588.40$$

The interest paid in the four months is the sum of the Interest columns.
Total = $1593.75 + $1591.98 + $1590.19 + $1588.40 = $6364.32

SECTION II

Question 23

(a) Deductions = $350 + $2000 + $250 = $2600
Taxable income = $56 350 − $2600 = $53 750
Medicare Levy = 1.5% of $53 750 = 0.015 × $53 750 = $806.25

(b) (i)

(ii) From the pattern, $s = 3n + 2$

$$\therefore s = 3n + 2 = 3 \times 100 + 2 = 302 \text{ sticks.}$$

(iii)

$$543 = 3n + 2$$
$$541 = 3n$$
$$n = 541 \div 3 = 180\frac{1}{3}$$

But n should be a whole number.
∴ It is not possible to create a pattern using exactly 543 sticks.

(c) 10% pa converts to 5% interest per period (every 6 months) and 6 periods (3 years × 2 periods per year):
Compounded value of $1 = $1.340
Value of investment = $5000 × 1.340 = $6700.

(d) (i) Volume = 10 000 L ÷ 1000 L/m^3 = 10 m^3.

(ii)

$$\text{Area}_{\text{ellipse}} = \pi ab = \pi \times (1.5 \div 2) \times (1.34 \div 2) = 1.57865\ldots\text{ m}^2$$

Volume = Ah

$$10 = 1.57865\ldots \times h$$
$$h = 10 \div 1.57865\ldots = 6.3345\,m = 6.33\text{ m.} \quad \text{(or 633 cm)}$$

Question 24

(a) (i) Using the plan the, the width of the stairwell is 900 mm.

(ii) Using the plan, the dimensions are 2000 mm by 2000 mm.

(iii) From the plan:

$AB = 3600 + 90 + 2000 + 90 + 3915$
$= 9695 \text{ mm}$

OR

$AB = 3690 + 6485 - 240 - 240$
$= 9695 \text{ mm}$

(iv) By actual measurement:
20mm on the plan represents 2000mm.
$\therefore$ The scale is 1:100.
Width of window on plan is 18 mm again by measurement.

$\text{Actual width} = 18 \text{ mm} \times 100$
$= 1800 \text{ mm}$

(b) (i) The die has been rolled 72 times.

$72 = 16 + 11 + A + 8 + 12 + 15$
$72 = 62 + A$
$A = 10$

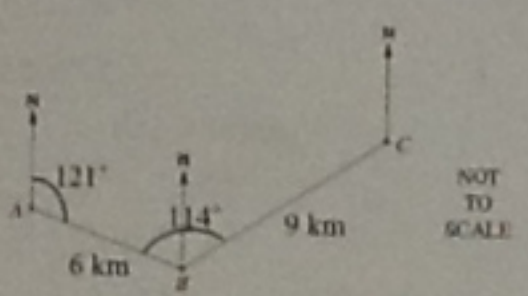
(ii) Relative frequency of obtaining a 4

$$\text{Relative frequency} = \frac{8}{72} = \frac{1}{9} \quad (\text{or } 11.\dot{1}\%)$$

(iii) Expected number of rolls for each number is $\frac{1}{6} \times 72 = 12$

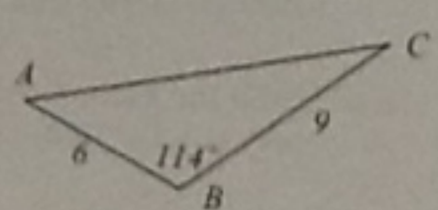
$\therefore$ '5' was rolled the expected number of times.

(c) (i)



$\angle ABN = 180° - 121° = 59°$
(co-interior angles)
$\angle NBC = 114° - 59° = 55°$
$\therefore$ Bearing of C from B is 055°T.

(ii) In ΔABC:



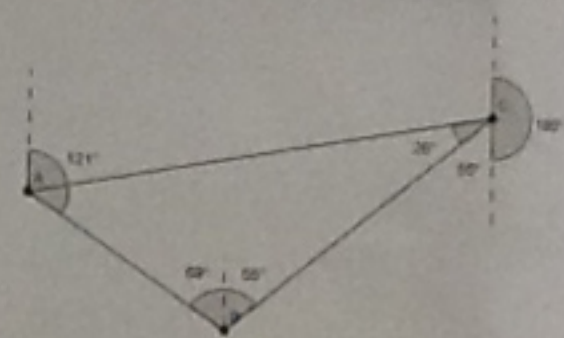
By cosine rule:

$b^2 = a^2 + c^2 - 2ac\cos B$
$b^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \times \cos 114°$
$b^2 = 160.9275575$
$b \approx 12.6857... \text{ km}$

$\therefore$ Distance AC is 13 km.

(iii) By sine rule:

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin 114°}{12.6857...} = \frac{\sin C}{6}$$
$$\sin C = \frac{6 \times \sin 114°}{12.6857}$$
$$\sin C = 0.43208...$$
$$\angle C = 26° \quad (\text{nearest degree})$$



Bearing of A from C is
$180° + 55° + 26° = 261°\text{T}$

NB If 13 km from part (ii) is used instead of the more accurate answer, then the bearing is 260°T (by sine rule) or 259°T (by cosine rule).

Question 25

(a) (i) Data is categorical.

(ii) This would be any question that would call for a numerical response, for example, "How many phone calls do you make per day?" or "What is your average monthly phone bill?"

(iii) Work out the exact proportion of the total number of students each academic year is, and ensure the same proportion is used in the survey. That is, if you have 8000 Year 12 students out of 80 000 in the entire state, you

will need exactly $\frac{1}{10}$ of your survey respondents to be from Year 12 for your survey to be stratified.

(iv) For a census you would need to survey all NSW high school students. (i.e. the entire 'population' is surveyed.)

(b) (i) Year 12 has the highest percentage of mobile phone owners, namely 100%.

(ii) Probability of owning a mobile phone:

Year 9: $\text{P(mobile)} = \frac{55}{70} \times 100$
$\approx 78.6\%$

Year 10: $\text{P(mobile)} = \frac{50}{60} \times 100$
$\approx 83.3\%$

A Year 10 student is more likely to own a mobile phone.

(iii) The trend is that the percentage of mobile phone ownership increases with year level.

(c) (i) $\text{Total number} = 330 + 250$
$= 580$

<u>OR</u>

$\text{Total number} = 319 + 261$
$= 580$

(ii) Females only:
$\text{P(prepaid)} = \frac{172}{319}$

(iii) There are now 113 males on a plan, out of a new total of 271 males.
Males only:
$\text{P(prepaid)} = \frac{113}{271}$
$= \frac{113}{271} \times 100$
$\approx 42\%$ (nearest %)

(d) (i) Outlier is 71.

(ii) For the female data, the scores are:
8 9 11 ***11*** 12 15 16
18 18 18 ***20*** 21 27 34

$Q_2 = \frac{16+18}{2} = 17$

Q_2 breaks the scores into 2 lists. The middle of each of the two lists is in bold, giving $Q_1 = 11$ and $Q_3 = 20$.

$\text{Interquartile range} = Q_3 - Q_1$
$= 20 - 11$
$= 9.$

Question 26

(a) (i) $X = 3 + 2 = 5.$

(ii) $P(<4) = \frac{6}{12} = \frac{1}{2}.$ (or 50%)

(iii) When 2 is obtained on B, the possible sums are 3, 3 or 5.
$\therefore P(3) = \frac{2}{3}$ (or $66.\dot{6}\%$)

(iv) $\text{Expectation} = 12 \times \frac{4}{12} + 0 \times \frac{6}{12} - 3 \times \frac{2}{12} - 5$
$= 4 + 0 - 0.5 - 5$
$= -1.5$
i.e. expect to lose \$1.50

(b) (i) Try $t = 6$:
$5 \times 3^6 = 3645$
Thus t = 6 is too small.

(ii) Try $t = 7$:
$5 \times 3^7 = 10\,935$
So t = 7 is too small.

Try $t = 8$:
$5 \times 3^8 = 32\,805$
Thus t = 8 is too big.

$\therefore$ It will take until 8 years for the population to exceed 18 000.

(c) $\text{Deposit} = 15\% \text{ of } \$20\,000$
$= 0.15 \times \$20\,000$
$= \$3000$
$\text{Balance owing} = \$20\,000 - \$3000$
$= \$17\,000$

$$\text{Interest} = Prn$$
$$= \$17\,000 \times 0.19 \times 5$$
$$= \$16\,150$$
$$\text{Total owed} = \$17\,000 + \$16\,150$$
$$= \$33\,150$$
$$\text{Repayment} = \$33\,150 \div (5 \times 12)$$
$$= \$552.50$$

Question 27

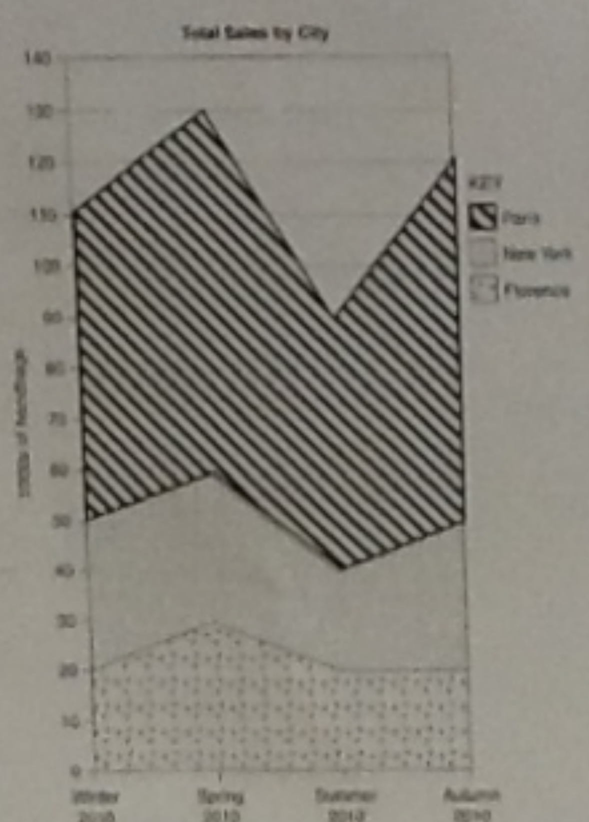
(a) Using values from the given graph:

Winter:
$\$50\,000 + \$60\,000 = \$110\,000$

Spring:
$\$60\,000 + \$70\,000 = \$130\,000$

Summer:
$\$40\,000 + \$50\,000 = \$90\,000$

Autumn:
$\$50\,000 + \$70\,000 = \$120\,000$



(b) (i) Pontianuk has a longitude of 109°E i.e. it is 71° from the dateline.
Jarvis Island has a longitude of 160°W i.e. it is 20° from the dateline
Angular distance is 71° + 20° = 91°
As both lie on the equator,

$$l = \frac{\theta}{360} \times 2\pi r$$
$$= \frac{91}{360} \times 2 \times \pi \times 6400$$
$$= 10\,165 \text{ km}$$

(ii) Rabaul is 4° south of Jarvis Island.
$\therefore$ latitude is: 4°S.
Rabaul is 48° west of Jarvis Island.
longitude: 160°W + 48°W = 208°W
But 180°W is the limit. This is 28° past this limit.
longitude: 180°E 28° = 152°E
$\therefore$ Rabaul is 4°S, 152°E.

(c) (i)
$$z = \frac{x - \bar{x}}{s}$$
$$= \frac{400 - 500}{50}$$
$$= \frac{-100}{50}$$
$$= -2$$

(ii) For 400 hours with Brand A:
$$z = \frac{x - \bar{x}}{s}$$
$$= \frac{400 - 450}{25}$$
$$= \frac{-50}{25}$$
$$= -2$$

Brand B: $z = -2$ (calculated above)
Both brands have the same z-score of 2 standard deviations below the mean for 400 hours. Therefore, the brands are equally likely to be defective. Therefore the statement is incorrect.

(d) Josephine:
$$A = P(1 + r)^n$$
$$= 50\,000(1 + 0.06)^{15}$$
$$= \$119\,827.91$$
$$\text{Gain} = \$119\,827.91 - \$50\,000$$
$$= \$69\,827.91$$

Emma:

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$= 500\left\{\frac{(1+0.005)^{180} - 1}{0.005}\right\}$$

$$= \$145\ 409.36$$

$$\text{Gain} = \$145\ 409.36 - \$90\ 000$$

$$= \$55\ 409.36$$

$\therefore$ Josephine has the better financial gain (by \$14 418.55).

Question 28

(a) (i) $P \propto \frac{1}{V}$

$$P = \frac{a}{V}$$

(ii) Substituting: P = 3, V = 2

$$3 = \frac{a}{2}$$

$$a = 3 \times 2$$

$$= 6$$

Thus $P = \frac{6}{V}$

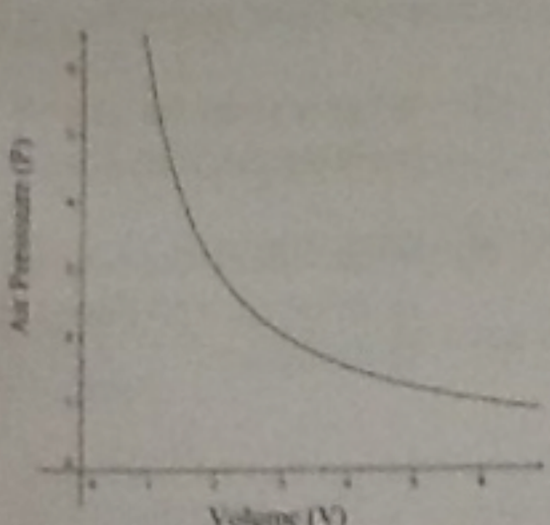
When V = 4:

$$P = \frac{6}{4}$$

$$= 1.5$$

(iii) Using easy values gives:

V	1	2	3	6
P	6	3	2	1



(b) (i) The line passes through (0, 60 000) and (15, 0):

$$m = \frac{0 - 60000}{15 - 0}$$

$$= \frac{-60000}{15}$$

$$= -4000$$

(ii) The gradient represents the depreciation rate or annual decline in the value of the tractor. i.e. it decreases by \$4 000 per year.

(iii) $y = mx + b$

$$S = -4000n + 60000$$

(iv) The values that are not suitable are: $n < 0$, negative values for time, and $n > 15$, time after the salvage value has reached \$0.

(v) $S = V_0(1-r)^n$

$$= 60000(1-0.20)^{14}$$

$$= \$2638.83$$

(vi) Consider values of n greater than 15:

For $n = 16$,

$$S = V_0(1-r)^n$$

$$= 60000(1-0.20)^{16}$$

$$= \$1688.85$$

For $n = 20$,

$$S = V_0(1-r)^n$$

$$= 60000(1-0.20)^{20}$$

$$= \$691.75$$

The value continues to decline and approaches zero, but never reaches it.

End of General Mathematics solutions